Working With Numbers Teaching Aids

Number Facts

TEACHER'S INSTRUCTIONS

By

JOYCE BENBROOK and CECILE FOERSTER

The Steck Company, *Publishers* Austin, Texas

TABLE OF CONTENTS

INTRODUCTION

These directions provide a wide variety of suggestions for directing the experiences of children which will lead to the development of meaningful number concepts. Emphasis is placed on building understandings of numbers and processes before providing practice to develop skills. At any grade level, as in grades one and two, much use is made of concrete objects and semi-concrete representations of numbers to build desired concepts and understandings.

The children are not asked at any time to memorize facts which they do not understand or to acquire skills in which they see no sense. Skill is developed through practice with examples and problems which are introduced according to relationships and in the order of their difficulty. It is the responsibility of the teacher to see that no new step of difficulty is presented until understanding and mastery of the preceding step are achieved.

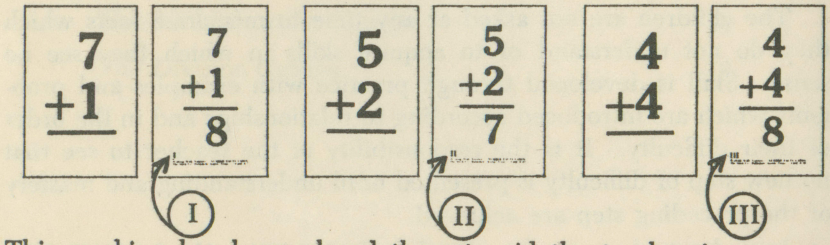
In order to lay an adequate foundation in numbers, slow, careful instructions are necessary. Not only the activities mentioned in *Teacher's Instructions* should be used but many other similar ones. Careful guidance and supervision by the teacher are of utmost importance.

FLASH CARDS

The 390 basic number facts in addition, subtraction, multiplication, and division are included in this set of cards. The addition, subtraction, and multiplication sets each contain 100 cards, and the division set contains 90 cards.

ADDITION AND SUBTRACTION CARDS

The addition and subtraction cards are each divided into six groups according to the generalizations which are to be developed in teaching them. Each card is marked on the back with Roman numerals I to VI to indicate the group to which it belongs, as:



This marking has been placed there to aid the teacher in separating the groups for teaching and practice exercises. Some cards will have the numbers of two or three groups on them. This indicates that the fact belongs to more than one group. After mixed drill exercises the cards can be quickly sorted by a child as well as the teacher and be ready for use in teaching related groups of facts.

THE ADDITION GROUPS

Group I. In Relation to Counting by 1's. Cards in this group are marked with Roman numeral I. (17 facts)

1	2	3	4	5	6	7	8	9
+1	+1	+1	+1	+1	+1	+1	+1	+1

1	1	1	1	1	1	1	1
+2	+3	+4	+5	+6	+7	+8	+9

Teach that:

1. Adding 1 to a number is equivalent to saying the next number when counting.
2. Changing the places of the numbers in an addition fact does not change the sum. That is, if 6 and 1 are 7, then 1 and 6 are 7.

Group II. In Relation to Counting by 2's. Cards in this group are marked with Roman numeral II. (15 facts)

2	3	4	5	6	7	8	9
+2	+2	+2	+2	+2	+2	+2	+2

2	2	2	2	2	2	2
+3	+4	+5	+6	+7	+8	+9

Teach that:

1. Adding 2 to a number is equivalent to skipping a number in the counting series just as counting by 2's is equivalent to skipping a number.

2. If 5 and 2 are 7, then 2 and 5 are 7.

$\frac{1}{2}$ and $\frac{2}{1}$ are facts introduced in Group I which also fit into this group. They have three symbols on the back to indicate that they should be included in three groups for study.

Group III. In Relation to the "Doubles" Facts. Cards in this group are marked with Roman numeral III. (19 facts)

3	3	4	4	4	5	5	5	6
+3	+4	+3	+4	+5	+4	+5	+6	+5

6	6	7	7	7	8	8	8	9	9
+6	+7	+6	+7	+8	+7	+8	+9	+8	+9

In the first group above, for example. teach that:

1. As 4 is 1 more than 3, the sum of 3 and 4 must be 1 more than the sum of 3 and 3.
2. If you know the sum of 3 and 4, you know the sum of 4 and 3.

These six facts from Groups I and II should be included with this group for practice:

1	1	2	2	2	3
+1	+2	+1	+2	+3	+2

Group IV. In Relation to Zero. Cards in this group are marked with Roman numeral IV. (19 facts)

0	1	2	3	4	5	6	7	8	9
+0	+0	+0	+0	+0	+0	+0	+0	+0	+0

0	0	0	0	0	0	0	0	0
+1	+2	+3	+4	+5	+6	+7	+8	+9

Teach that:

1. Any number plus zero equals the number.
2. Zero plus any number equals the number.

Group V. In Relation to the "Tenness" of Our Number System. Cards in this group are marked with Roman numeral V. (10 facts)

9	9	9	9	9	3	4	5	6	7
+3	+4	+5	+6	+7	+9	+9	+9	+9	+9

Teach that:

1. Since 9 is 1 less than 10, 9 plus any number is 1 less than 10 plus that number.
2. The sum of 3 and 9 is the same as the sum of 9 and 3. These seven facts introduced in Groups I, II, and III should be included with this group for practice:

9	9	9	9	1	2	8
+1	+2	+8	+9	+9	+9	+9

Group VI. Miscellaneous Facts. Cards in this group are marked with Roman numeral VI. (20 facts)

5	6	7	6
+3	+3	+3	+4

3	3	3	4
+5	+6	+7	+6

8	8	8	8	7	7
+3	+4	+5	+6	+4	+5

3	4	5	6	4	5
+8	+8	+8	+8	+7	+7

Let the children tell how they recall these sums, as:

1. I know 5 and 3 are 8 because they are the same as 4 and 4.
2. I know 8 and 3 are 11 because 10 and 3 are 13, and 8 is 2 less than 10.

THE SUBTRACTION GROUPS

Group I. In Relation to Counting by 1's. Cards in this group are marked with Roman numeral I. (17 facts)

2	3	4	5	6	7	8	9	10
−1	−1	−1	−1	−1	−1	−1	−1	−1

3	4	5	6	7	8	9	10
−2	−3	−4	−5	−6	−7	−8	−9

Teach that:

1. When we take 1 from a number, it is equivalent to saying the number that comes just before it in the counting series.
2. When two numbers come right together in the counting series and the smaller is subtracted from the larger, the remainder will always be 1.

Group II. In Relation to Counting by 2's. Cards in this group are marked with Roman numeral II. (15 facts)

4	5	6	7	8	9	10	11
−2	−2	−2	−2	−2	−2	−2	−2

5	6	7	8	9	10	11
−3	−4	−5	−6	−7	−8	−9

Teach that:

1. Taking away 2 from a number is related to counting backward by 2's just as adding 2 is related to counting forward by 2's.
2. If 2 from 5 is 3, then 3 from 5 is 2, etc.

   $\begin{array}{r} 3 \\ -2 \\ \hline \end{array}$ and $\begin{array}{r} 3 \\ -1 \\ \hline \end{array}$ are facts introduced in Group I which also belong to this group and should be included with this group for practice.

Group III. In Relation to the "Doubles" Facts. Cards in this group are marked with Roman numeral III. (19 facts)

6	7	7	8	9	9	10	11	11
−3	−3	−4	−4	−4	−5	−5	−5	−6

12	13	13	14	15	15	16	17	17
−6	−6	−7	−7	−7	−8	−8	−8	−9

18
−9

In the first facts in Group III, for example, teach that:

1. Since 7 is 1 more than 6, 3 from 7 is 1 more than 3 from 6.
2. If 3 from 7 is 4, then 4 from 7 is 3.

These six facts from Groups I and II should be included with this group for practice:

2	3	3	4	5	5
−1	−1	−2	−2	−2	−3

Group IV. In Relation to Zero. Cards in this group are marked with Roman numeral IV. (19 facts)

0	1	2	3	4	5	6	7	8	9
−0	−0	−0	−0	−0	−0	−0	−0	−0	−0

1	2	3	4	5	6	7	8	9
−1	−2	−3	−4	−5	−6	−7	−8	−9

Teach that:

1. Zero subtracted from any number leaves that number.
2. Any number less itself leaves zero.

Group V. In Relation to the "Tenness" of Our Number System. Cards in this group are marked with Roman numeral V. (10 facts)

12	13	14	15	16
−9	−9	−9	−9	−9

12	13	14	15	16
−3	−4	−5	−6	−7

Teach that:

1. Since 9 is 1 less than 10, 9 from a teens number will always be 1 more than the number of 1's expressed in the teens number.
2. If 9 from 12 is 3, then 3 from 12 is 9, etc.

These seven facts introduced in Groups I, II, and III should be included with this group for practice:

10	11	17	18	10	11	17
−9	−9	−9	−9	−1	−2	−8

Group VI. Miscellaneous Facts. Cards in this group are marked with Roman numeral VI. (20 facts)

8	9	10	10
−3	−3	−3	−4

8	9	10	10
−5	−6	−7	−6

11	12	13	14	11	12
−3	−4	−5	−6	−4	−5

11	12	13	14	11	12
−8	−8	−8	−8	−7	−7

Let the children tell how they know these remainders, as:

1. 3 from 8 is 5 because 4 from 8 is 4, and 3 is 1 less than 4.
2. 8 from 12 is 4 because 8 and 2 are 10 and 2 more are 12. 2 and 2 are 4.

If the relationships suggested here are taught and followed by drill, automatic responses to the number facts will result. The roundabout way of thinking and talking about facts to ensure understanding will not become permanent.

Order of Teaching

After the addition facts in Group I are taught, teach the subtraction facts in Group I. Then teach the relationship of the facts, as:

8	1	9	9
+1	+8	−1	−8

Follow with drill in separate groups and in mixed groups.

Teach the addition facts in Group II. Follow with drill as a separate group. Then have mixed practice with the addition facts in Groups I and II.

Teach the subtraction facts in Group II. Note the relationship to the addition facts in Group II. Drill on these addition and subtraction facts as separate groups, then in mixed order with the facts in Group I.

Continue introducing the groups in this manner until all the facts have been taught and sufficient practice provided in order that mastery may be achieved.

The number fact with the answer is on the back of each card so that the cards can be used for independent study or for group study with a child leader.

MULTIPLICATION AND DIVISION CARDS

The multiplication and division cards are each divided into 3 groups and are marked with the Roman numerals I, II, and III on the reverse side of the cards. The cards should be introduced in this order, and near mastery of the facts in each group should be achieved before a new group is introduced. After considerable practice with the new group introduced, mixed practice with the cards from the two groups should follow. Likewise, after practice with the cards in Group III, then mixed practice with Groups I and II should follow.

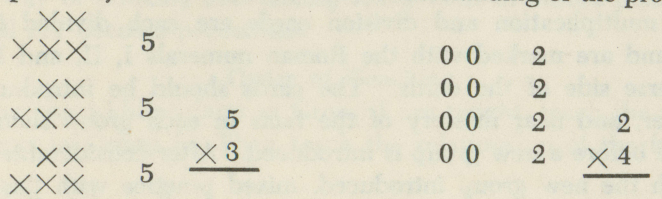
The reasons for arranging the multiplication facts in these 3 groups for study and practice are:

1. If the process of multiplication is to be made meaningful, it should be introduced in its relation to addition; that is, it is a short way to get a total when all the numbers to be combined are alike, as:

2	
2	2
2	×4
2	

This multiplication fact, for example, should be read "four 2's," and the pupils should understand and be able to say, "It means to take 2 four times." This understanding can be developed more readily by using the facts in Group I because the products are small, and children can visualize small numbers more easily than they can large numbers.

2. Concrete and semi-concrete representations of the facts in Group I can be made readily, thus giving the pupils the opportunity to demonstrate their understanding of the process, as:



3. All of the facts in Group II involve either a 1 or a 0 and should not be introduced until such time as there is a real need for them; this need arises when 2-place and 3-place multipliers are introduced, as:

41	60	421	301	200
×3	×2	×4	×5	×6

Children never meet the zero combinations and those involving 1 as the multiplier in isolation in real life situations; therefore, they should be taught in connection with example and problem-solving situations which are real to children. This makes necessary the teaching of several facts (those in Group I) before the zero and 1 facts can be introduced.

4. Multiplication facts should be introduced in the order of the size of the products and later organized in table form to show the relationship which exists. In other words, children should discover that a relationship between certain facts exists which can be shown by writing the facts in table form. They should make generalizations about these related facts which will aid in learning them.

THE MULTIPLICATION GROUPS

Group I. Products from 4 to 20. Cards in this group are marked with Roman numeral I. (25 facts)

2	3	2	4	2	3	5	2	6
×2	×2	×3	×2	×4	×3	×2	×5	×2
2	4	3	7	2	5	3	8	2
×6	×3	×4	×2	×7	×3	×5	×2	×8
4	9	2	6	3	5	4		
×4	×2	×9	×3	×6	×4	×5		

Group II. Facts involving 1 and 0. Cards in this group are marked with Roman numeral II. (36 facts)

1	2	3	4	5	6	7	8	9
×1	×1	×1	×1	×1	×1	×1	×1	×1
1	1	1	1	1	1	1	1	0
×2	×3	×4	×5	×6	×7	×8	×9	×0
0	0	0	0	0	0	0	0	0
×1	×2	×3	×4	×5	×6	×7	×8	×9
1	2	3	4	5	6	7	8	9
×0	×0	×0	×0	×0	×0	×0	×0	×0

Group III. Products from 21 to 81. Cards in this group are marked with Roman numeral III. (39 facts)

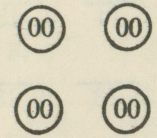
7	3	8	3	6	4	5	9	3
×3	×7	×3	×8	×4	×6	×5	×3	×9
7	4	6	5	8	4	7	5	9
×4	×7	×5	×6	×4	×8	×5	×7	×4
4	6	8	5	7	6	9	5	8
×9	×6	×5	×8	×6	×7	×5	×9	×6
6	7	9	6	8	7	9	7	8
×8	×7	×6	×9	×7	×8	×7	×9	×8
9	8	9						
×8	×9	×9						

The reasons for arranging the division facts in these three groups for study and practice are:

1. If the process of division is to be made meaningful, it should be introduced in its relation to successive subtraction. For example, children should use concrete materials to prove that there are four 2's in 8 by taking away 2 objects at a time until they are all gone and noting that 4 groups of 2 objects had to be taken away. This understanding can be developed more readily by using the facts in Group I because the dividends are small. Later, the relation of division to finding a fractional part should be taught.

2. Because of the size of the dividends in Group I, a picture of the division fact can be readily made. This is exceedingly important in the initial stage of developing understanding and skill in division. Give directions similar to the following:

   "Make 8 circles on your paper. Draw rings around 2 circles at a time until all the circles are in rings. Write the division fact your picture represents."



$$2\overline{)8}\quad 4$$

   Read: How many 2's are there in 8? The answer is 4.
3. All the facts in Group II involve either a 1 or a 0 and should not be introduced until such time as there is a real need for them. The needs arises when 2 and 3-place dividends are introduced, as:

$$2\overline{)80} \qquad 4\overline{)48} \qquad 3\overline{)630} \qquad 5\overline{)55}$$

4. The relationship between multiplication and division should be taught, and the division facts should be grouped in the same manner as the multiplication facts to ensure the proper teaching of this relationship.
5. Division facts should be introduced in the order of the size of the dividends and later organized in table form to aid in learning them.

THE DIVISION GROUPS

Group I. Dividends from 4 to 20. Cards in this group are marked with Roman numeral I. (25 facts)

$2\overline{)4}$ $2\overline{)6}$ $3\overline{)6}$ $2\overline{)8}$ $4\overline{)8}$ $3\overline{)9}$ $2\overline{)10}$ $5\overline{)10}$

$2\overline{)12}$ $6\overline{)12}$ $3\overline{)12}$ $4\overline{)12}$ $2\overline{)14}$ $7\overline{)14}$ $3\overline{)15}$ $5\overline{)15}$

$2\overline{)16}$ $8\overline{)16}$ $4\overline{)16}$ $2\overline{)18}$ $9\overline{)18}$ $3\overline{)18}$ $6\overline{)18}$ $4\overline{)20}$

$5\overline{)20}$

Group II. Facts involving 1 and 0. Cards in this group are marked with Roman numeral II. (26 facts)

$1\overline{)1}$ $1\overline{)2}$ $1\overline{)3}$ $1\overline{)4}$ $1\overline{)5}$ $1\overline{)6}$ $1\overline{)7}$ $1\overline{)8}$

$1\overline{)9}$ $2\overline{)2}$ $3\overline{)3}$ $4\overline{)4}$ $5\overline{)5}$ $6\overline{)6}$ $7\overline{)7}$ $8\overline{)8}$

$9\overline{)9}$ $1\overline{)0}$ $2\overline{)0}$ $3\overline{)0}$ $4\overline{)0}$ $5\overline{)0}$ $6\overline{)0}$ $7\overline{)0}$

$8\overline{)0}$ $9\overline{)0}$

Group III. Dividends from 21 to 81. Cards in this group are marked with Roman numeral III. (39 facts)

$3\overline{)21}$ $7\overline{)21}$ $3\overline{)24}$ $8\overline{)24}$ $4\overline{)24}$ $6\overline{)24}$ $5\overline{)25}$

$3\overline{)27}$ $9\overline{)27}$ $4\overline{)28}$ $7\overline{)28}$ $5\overline{)30}$ $6\overline{)30}$ $4\overline{)32}$

$8\overline{)32}$ $5\overline{)35}$ $7\overline{)35}$ $6\overline{)36}$ $4\overline{)36}$ $9\overline{)36}$ $5\overline{)40}$

$8\overline{)40}$ $6\overline{)42}$ $7\overline{)42}$ $5\overline{)45}$ $9\overline{)45}$ $6\overline{)48}$ $8\overline{)48}$ $7\overline{)49}$

$6\overline{)54}$ $9\overline{)54}$ $7\overline{)56}$ $8\overline{)56}$ $7\overline{)63}$ $9\overline{)63}$ $8\overline{)64}$ $8\overline{)72}$

$9\overline{)72}$ $9\overline{)81}$

The number fact with the answer is on the back of each card so that they can be used for independent study or for group study with a child leader.