

Working With Numbers Teaching Aids

FOR THE UPPER ELEMENTARY GRADES

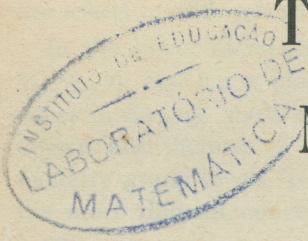
C A R D S F O R

BUILDING THE MEANING OF FRACTIONS

Set 1 — MEANING OF PARTS OF A WHOLE

Set 2 — MEANING OF PARTS OF A GROUP

TEACHER'S MANUAL



By

JOYCE BENBROOK and CECILE FOERSTER

The Steck Company publishers Austin, Texas

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INTRODUCTION

It is very important that the meaning of fractions be developed before computation involving fractions is introduced. If adequate concepts are not built, children will be forced to perform operations according to rules which they have memorized but seldom understand. Such rules are difficult to recall and apply after direct teaching of them ceases, and they are frequently confused during the period of direct teaching. Therefore, it is necessary that many experiences with fractions using concrete and semi-concrete materials and extending over a long period of time be provided before boys and girls are asked to add, subtract, multiply, or divide them.

In the lower elementary grades work with unit fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. has a place in the arithmetic program. Many experiences in finding "parts of a whole" and "parts of a group" using real objects and pictures of objects will be necessary. At the upper elementary grade levels "reteaching" of unit fractions is in order before the more difficult concepts are introduced.

The criteria to be used in judging the depth of a child's understanding of a number concept is his ability to:

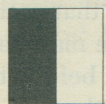
1. Identify a situation — to recognize that $\frac{1}{4}$ of a pie remains uneaten, $\frac{2}{3}$ of a circle is colored, $\frac{3}{4}$ of a square is more than $\frac{5}{8}$ of a square of equal size, etc.
2. Reproduce a situation — to measure $\frac{2}{3}$ of a cup of sugar, to cut $\frac{1}{4}$ of a yard of ribbon, to shade $\frac{3}{5}$ of a circle, etc.

The two sets of fraction flash cards in the *Working With Numbers Teaching Aids* series published by The Steck Company, Austin, Texas, will provide all the semi-concrete material needed for developing the ability to identify situations which exhibit the fractional relationships that must be understood before computation with fractions is introduced. Additional experiences with concrete and semi-concrete materials will have to be provided by the teacher to check the pupil's ability to reproduce fractional situations.

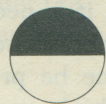
SUGGESTIONS FOR BUILDING THE MEANING OF FRACTIONAL PARTS OF A WHOLE

The 60 cards included in this group of fraction cards are reproduced here in an arrangement that provides a quick overview of all the cards and suggests many ways for using them in building the fundamental concepts about fractional parts of a whole.

HALVES



1



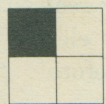
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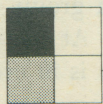
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Cards

FOURTHS



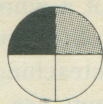
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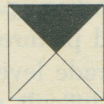
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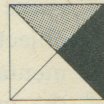
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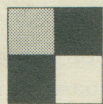
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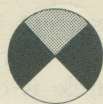
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9



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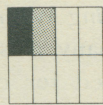


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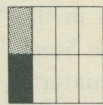
EIGHTHS



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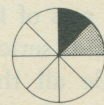
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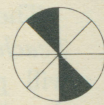
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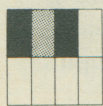
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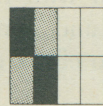
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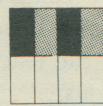
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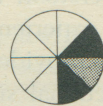
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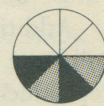
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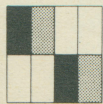
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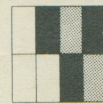
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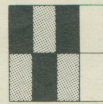
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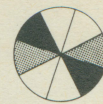
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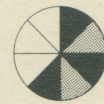
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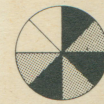
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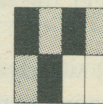
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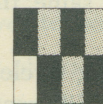
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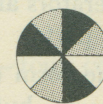
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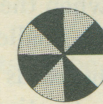
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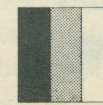
THIRDS



2



3



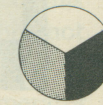
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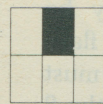


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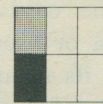


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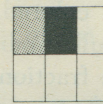
SIXTHS



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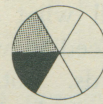
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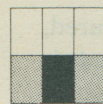
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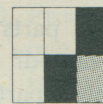
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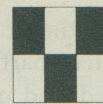
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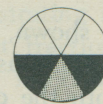
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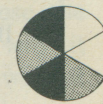
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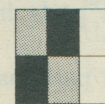
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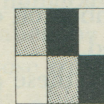
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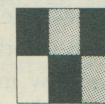
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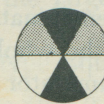
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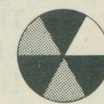
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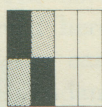
These cards can and should be grouped in many different ways as they are being used in the classroom. All the cards in the pack should not be used in the introductory stages of developing an understanding.

Suggested grouping:

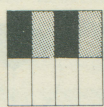
1. Unit fractions — $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ — to emphasize the fact that a unit fraction is one of the equal parts that a whole may be divided into. (An additional concept to be developed is that fourths, etc. of the same size objects are equal although not the same shape.)
2. All the cards showing eighths — to emphasize the fact that when the denominators are the same, the larger the numerator the larger the part.
3. All the cards with numerators of 1 — to emphasize the fact

that when the numerators are the same, the larger the denominator the smaller the part.

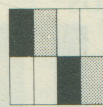
4. All the cards of one form (square, circle, etc.) which show the same fractional parts but in different arrangements. For example: $\frac{1}{8}$ is shown in 3 different arrangements.



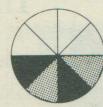
(Card 23)



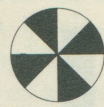
(Card 24)



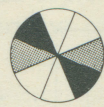
(Card 25)



(Card 54)



(Card 55)



(Card 56)

I. Recognition — reading, writing, and meaning.

- A. Tell what fractional part is colored. The cards may be placed in wall pockets, on bulletin boards, or used as flash cards. Both the name of the fraction and its meaning must be emphasized: one half is its name, and it means 1 of the 2 equal parts.
- B. As each card is shown, write the fractional parts pictured, using such directions as: What part is colored? What is pictured with both the colored and uncolored parts?
- C. Introduce the terms denominator and numerator.
1. The denominator shows the number of equal parts into which the whole has been divided.
 2. The numerator shows how many of these equal parts have been taken.

II. Equivalent fractions

Group the cards to show equivalent fractions as, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{3}{6} - \frac{1}{3}$, $\frac{2}{6} - \frac{1}{4}$, $\frac{2}{8}$. The pupils will discover that $\frac{4}{8}$ is the same as $\frac{1}{2}$, etc. From this type of experience generalizations about reducing fractions to lower terms and raising them to higher terms should be developed. Questions similar to the following will focus attention on the relationships which must be noted before the generalizations can be stated:

1. The denominator is how much larger than the numerator in the fraction $\frac{1}{2}$? $\frac{2}{4}$? $\frac{4}{8}$? $\frac{3}{6}$? $\frac{1}{3}$? $\frac{2}{6}$?
2. What is the relationship between the numerators in the paired parts below? Between the denominators?

$$\frac{1}{2} \text{ and } \frac{2}{4}$$

$$\frac{1}{2} \text{ and } \frac{4}{8}$$

$$\frac{1}{2} \text{ and } \frac{3}{6}$$

$$\frac{2}{4} \text{ and } \frac{4}{8}$$

$$\frac{1}{3} \text{ and } \frac{2}{6}$$

$$\frac{1}{4} \text{ and } \frac{2}{8}$$

3. Could a rule be made about changing fractions to higher terms? To lower terms?

III. Comparative size of fractions — four concepts to be developed with the cards.

- A. $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{6}{6}$, $\frac{8}{8}$ equal one whole. Use any and all of the cards to illustrate that the part colored plus the part not colored on each cards is $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{6}{6}$, or $\frac{8}{8}$ and that this is a whole. Extend this understanding to other fractions not pictured on the cards.
- B. If the denominators are the same, the larger the numerator the larger the part.
1. Use cards of one form (square or circle) to show this progressive increase in size. Place them in order in a wall pocket or on a bulletin board — $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$.
 2. Have pupils choose cards with like denominators and arrange them in decreasing as well as increasing size.
 3. From several cards illustrating like fractions, choose the one showing the largest fractional part and tell why it is largest; the smallest fractional part and tell why it is smallest.
- C. If the numerators are the same, the larger the denominator the smaller the part.
1. Choose from the cards of one type (square or circle) those cards which show $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$; $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{6}$, and $\frac{2}{8}$; $\frac{3}{4}$, $\frac{3}{6}$, and $\frac{3}{8}$; $\frac{4}{6}$ and $\frac{4}{8}$; $\frac{5}{6}$ and $\frac{5}{8}$. Arrange the cards of each group according to size and write the fractional part that each card shows. (A parallel activity would be to require pupils to illustrate fractional parts and arrange them according to size.) This emphasizes again the meaning of the numerator and the denominator of a fraction that has previously been introduced.
 2. Have children choose cards with like numerators and arrange them according to both increasing and decreasing size.
 3. From several cards illustrating the same number of colored parts, choose the one showing the largest fractional part and tell why it is the largest; the smallest fractional part and tell why it is the smallest.

- D. If the denominators of fractions are unlike, they must be

changed to like fractions before an exact comparison can be made.

1. Place cards illustrating $\frac{1}{2}$ and $\frac{3}{8}$ in a wall pocket or in the chalk tray. Substitute the card showing $\frac{1}{8}$ for the card showing $\frac{1}{2}$. Recall the generalization developed in relation to raising fractions to higher terms and help children to formulate a rule for finding the least common denominator. For example:

- To change a fraction to higher terms, multiply both the numerator and the denominator by the same number.
- When the denominators of two fractions are unlike, if the larger number can be divided evenly by the smaller number, it is the least common denominator. (The number to be used in raising a fraction to higher terms is found by dividing the larger denominator by the smaller.)

2. Use cards in pairs showing $\frac{1}{2}$ and $\frac{5}{6}$; $\frac{1}{3}$ and $\frac{5}{6}$; and $\frac{1}{3}$ and $\frac{1}{6}$. Substitute cards showing $\frac{3}{6}$ and $\frac{2}{6}$ for the cards showing $\frac{1}{2}$ and $\frac{1}{3}$, respectively.

3. Use cards for $\frac{1}{2}$ and $\frac{4}{8}$; $\frac{2}{4}$ and $\frac{4}{8}$. Substitute cards showing $\frac{4}{8}$ and $\frac{4}{8}$ for the cards showing $\frac{1}{2}$ and $\frac{2}{4}$ respectively.

4. Use cards for $\frac{1}{2}$ and $\frac{1}{3}$. Substitute cards showing $\frac{3}{6}$ and $\frac{2}{6}$ for the cards showing $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

A second rule for finding the least common denominator can be developed here — the least common denominator of two unlike fractions is sometimes the product of the two denominators.

IV. Use in relation to decimal fractions and per cents.

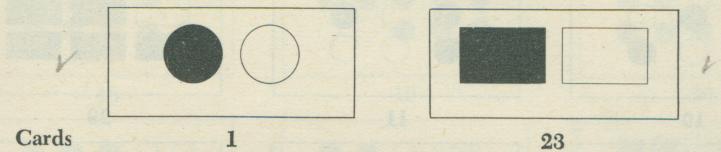
A. Cards picturing $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ can be used to illustrate decimal fractions after other materials have been used to develop the meaning of decimal fractions and to show their relationship to common fractions.

B. The most common per cents — 50%, 25%, 33 $\frac{1}{3}$ %, 75%, 12 $\frac{1}{2}$ %, 66 $\frac{2}{3}$ %, 87 $\frac{1}{2}$ %, 16 $\frac{2}{3}$ %, 83 $\frac{1}{3}$ % — can be illustrated with the cards.

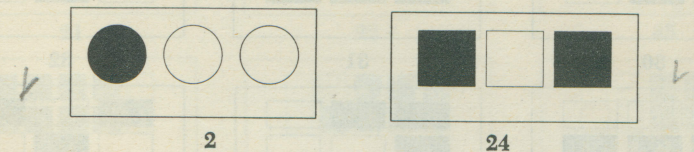
SUGGESTIONS FOR BUILDING THE MEANING OF FRACTIONAL PARTS OF A GROUP

The 50 cards included in this group are reproduced here in an arrangement that provides a quick overview of all the cards and suggests many ways for using them in building the fundamental concepts about fractional parts of a group.

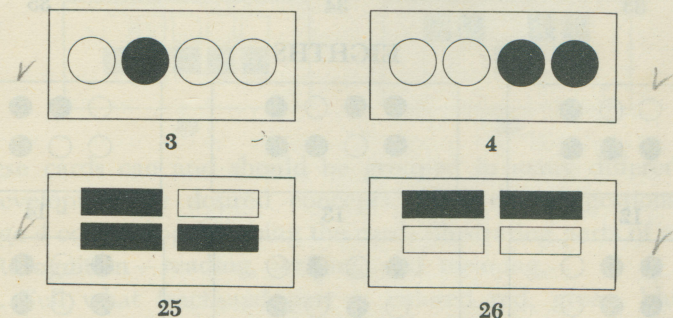
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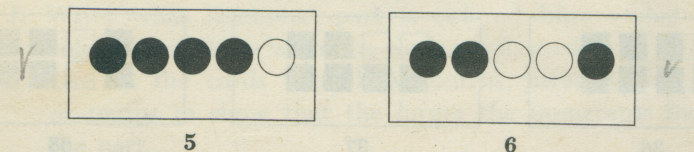
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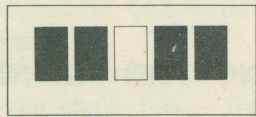


FOURTHS

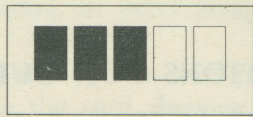


FIFTHS



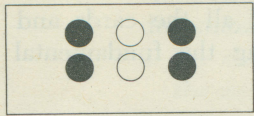


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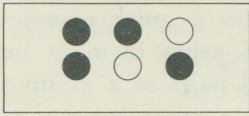


28

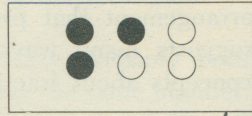
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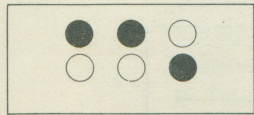
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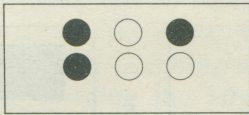
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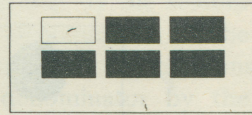
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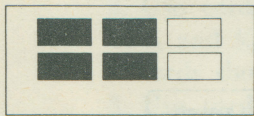
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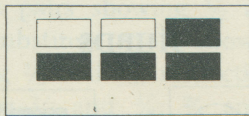
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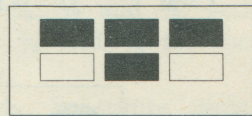
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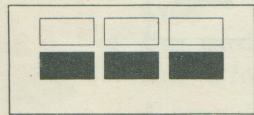
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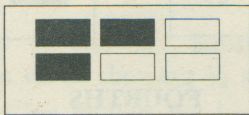
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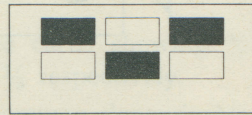
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33

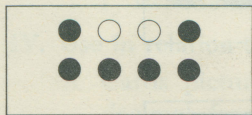


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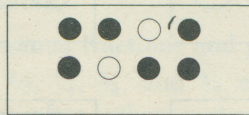


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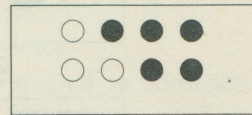
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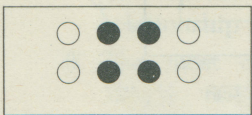
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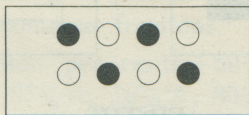
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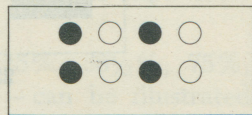
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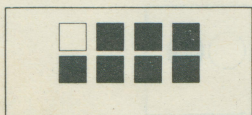
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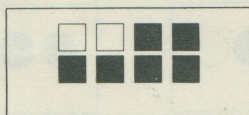
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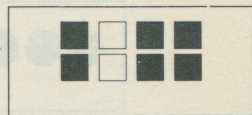
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36

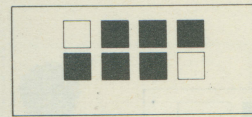


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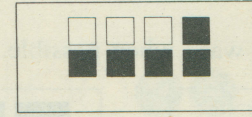


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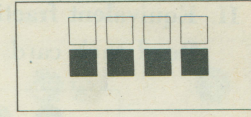
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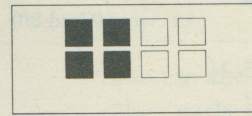
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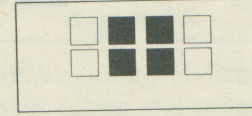
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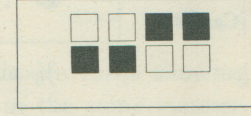
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42

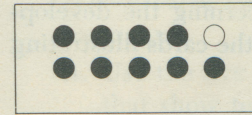


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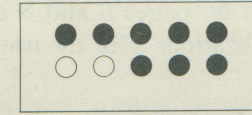


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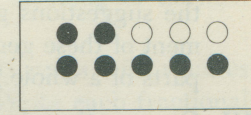
TENTHS



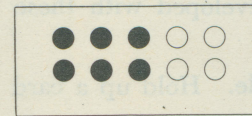
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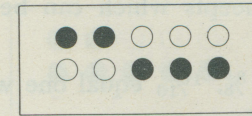
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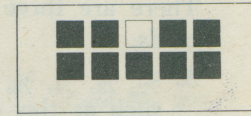
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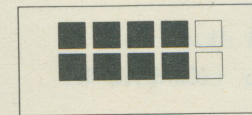
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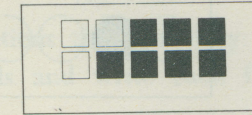
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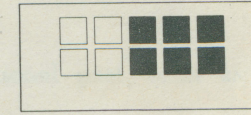
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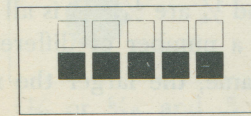
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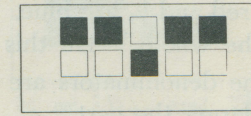
47



48



49



50

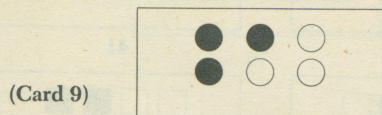
These cards can and should be grouped in many different ways for developing the desired concepts. See the suggestions given on page 4 concerning grouping the cards illustrating parts of a whole.

I. Recognition — reading, writing, and meaning.

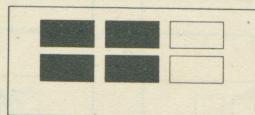
- A. Tell what fractional part is colored red, green, blue, etc. Emphasize the meaning of the fractional part as well as its name — "one-third means 1 of 3 equal parts," etc.
- B. Write what fractional part is colored blue — what part is colored red, etc. — when the cards are flashed.
- C. Use all the cards that show fourths, fifths, sixths, eighths, or tenths to show that the larger the numerator the larger the part.

II. Equivalent fractions

Read each card two ways when possible.



$\frac{3}{6}$ and $\frac{1}{2}$



(Card 30)

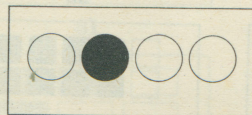
$\frac{2}{6}$ and $\frac{1}{3}$

From such experiences, generalizations about changing a fraction to higher and to lower terms should be developed. (See the suggestions given on pages 6 and 8 concerning the development of these generalizations with the use of the cards illustrating parts of a whole.)

III. Comparative size of fractions

There are three concepts which can be developed with these cards:

- A. $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{8}{8}, \frac{10}{10}$ equal one whole. Hold up a card such as this one:



(Card No. 3)

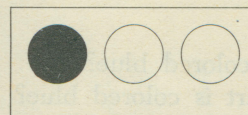
Help the child learn to describe it thus: " $\frac{3}{4}$ of the circles are red and $\frac{1}{4}$ are blue. $\frac{3}{4}$ and $\frac{1}{4}$ are $\frac{4}{4}$. $\frac{4}{4}$ is all the circles or the whole." Do this with a number of different cards.

- B. If the denominators are the same, the larger the numerator the larger the part.

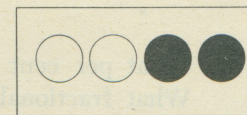
Choose all the cards illustrating sixths with squares, for example, and arrange them in increasing or decreasing size according to portions colored. Arrange other cards illustrating tenths, eighths, fifths, or fourths with circles or squares in the same manner until the concept is understood.

- C. If the numerators are the same, the larger the denominator the smaller the part.

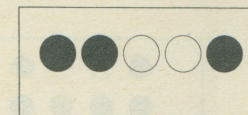
Choose all the cards using circles, for example, that show two of the fractional parts on each card and that show the two fractional parts in the same color —



(Card No. 2)



(Card No. 4)



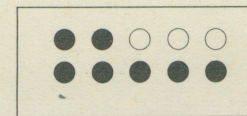
(Card No. 6)

Arrange the cards according to size. Choose cards showing other parts in other colors and proceed in the same manner.

IV. To show the relation of common fractions and decimal fractions.

(These cards can also be used in building meaning of decimal fractions.)

- A. Use the five cards with circles and the six cards with squares that show tenths (eleven cards in all). Write each fractional part two ways:



(Card No. 20)

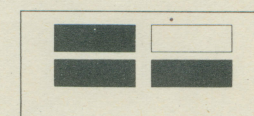
$\frac{3}{10}$ and .3

Read both fractions the same way.

- B. Use other cards and write each fractional part, such as:



$\frac{1}{2}$ or $\frac{5}{10}$ and .5

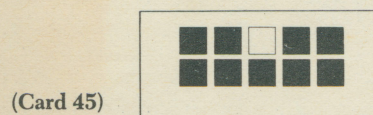


(Card 25)

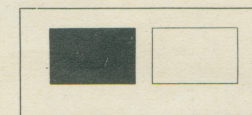
$\frac{3}{4}$ or $\frac{75}{100}$ and .75

- V. To show the relationship between common fractions, decimal fractions, and per cents. (These cards can also be used in building meaning of per cents.)

- A. By choosing cards in the same manner as suggested above, write the fractional parts illustrated in three different ways, such as:



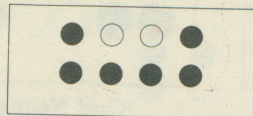
$\frac{1}{10}$.1 10%



(Card 23)

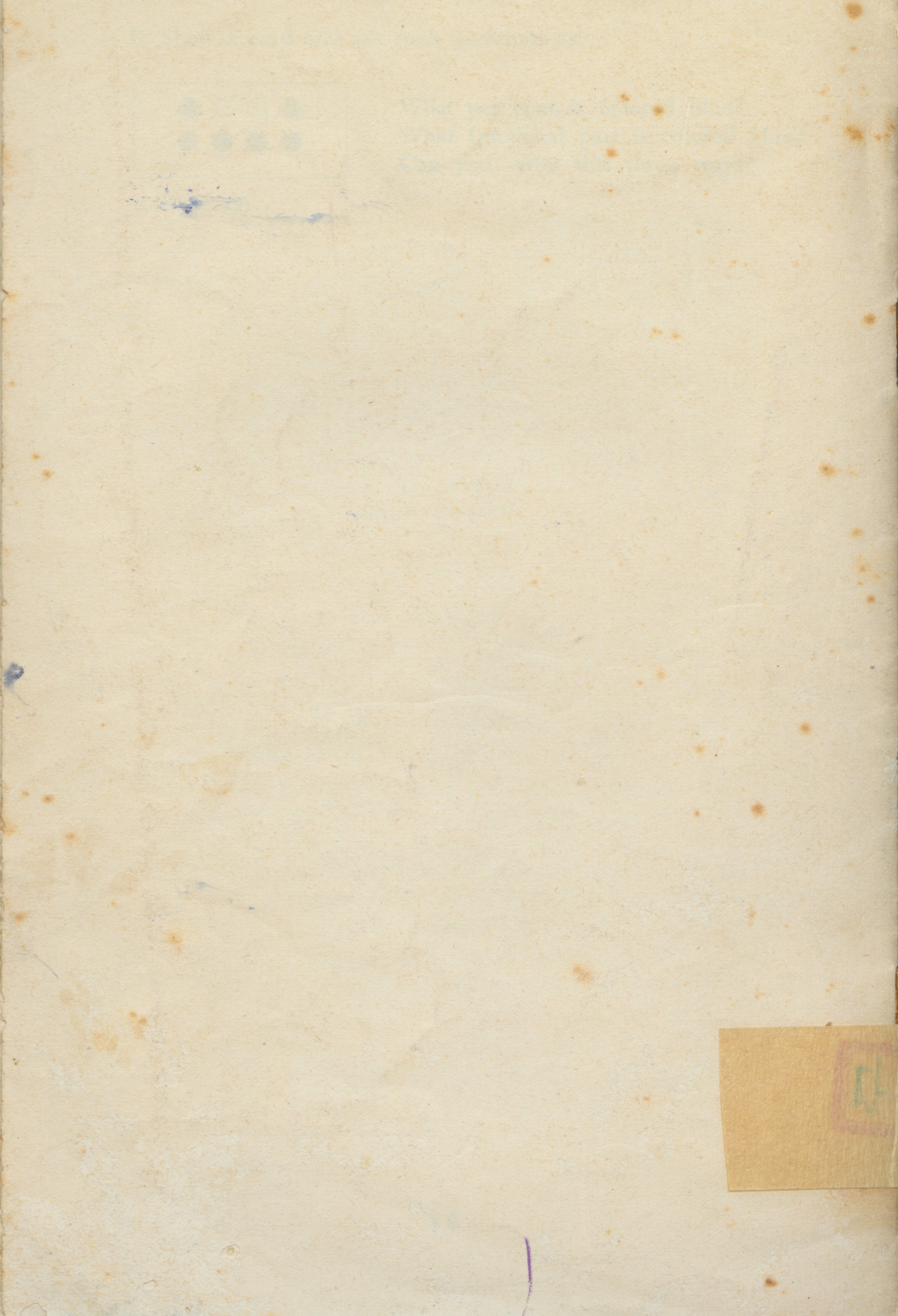
$\frac{1}{2}$.5 50%

B. Show a card and ask such questions as:



What per cent is colored blue?
What fractional part is colored blue?
Can you write this three ways?

(Card No. 12)



BENBROOK, N. Y.
Working with Numbers Teaching Aids.
UPPER ELEMENTARY GRADES
Cards for building the meaning of
fractions.
Teacher's Manual.
The Steck Co. Texas - 1950
14 pgs.

M